

**DEVELOPMENT OF A PREDICTIVE MODEL
FOR CHARACTERIZING THE RESILIENT RESPONSE
OF GRANULAR MATERIALS IN THE TROPICS**

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ABSTRACT

Most of the mathematical models available for characterizing the resilient response of granular materials for the analysis and design of pavement systems were developed for the temperate region. However, there are limitations in applying these models outside the developed jurisdiction especially in the tropical region. Therefore, a comparative study of these models is made with a goal to developing one for the tropical region. Based on correlation with field results collected from several randomly selected pavement sections in the tropical region, the widely used Smith and Witczak predictive model is selected for further study. Modifications of the algorithms of the model selected are made in-order to reflect the material non-linearity and to give a better fit to the resilient triaxial test data and fields measured values. Subsequently, the developed model gives a superior correlation with the field results obtained in the tropics. Influence of pavement variables on performance is determined from the model and a design chart for tropical roads also evolved from the study.

Keywords: Resilient Modulus, Non-linear, Granular Materials, and Characterization.

INTRODUCTION

The mechanistic design procedure used for the analysis and design of pavement systems involves the analysis of stresses and strains in the pavement structure and the determination of the allowable stresses and strains that the pavement materials can withstand. Such a procedure usually involves careful material characterization, proper stress analysis that take into account the complex stress dependent, nonlinear visco-elasto-plastic behavior of the materials, as well as a translation of the mathematical findings into performance prediction. Consequently this requires a proper material characterization of the main structural layer of the pavement which composes of mainly of

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granular materials.

A careful observation of the response of a granular soil sample under repeated loading in a triaxial test revealed that the soil tends to shake down to elastic response after a certain number of cycles at a fixed load level. At the initial stages the sample experiences inelastic deformations, but the amount of plastic flow decreases with cycling until the response is totally elastic. According to Hjelmstad and Taciroglu 2000, these observations have led researchers in the pavement community to hypothesize that granular materials shake down to resilient (elastic) behavior under repeated loading, and the nonlinear elasticity is a reasonable model of the behavior of granular materials that are used under pavements because of the compaction process used in constructing pavements is thought to provide a sufficient excitation to shake down the material relative to the loads that the complete pavement will see in service.

Consequently the focus of much effort to characterize the non-linear elastic response of granular materials have been to capture the stress dependence of the resilient modulus. One widely used form of expressing this nonlinearity is by use of:

$$M_R = \frac{\sigma_d}{\varepsilon_a} = K_1 \theta^{K_2} \quad (1)$$

where: M_R = resilient modulus, σ_d = axial deviator stress, ε_a = recoverable axial strain, θ = bulk stress (first stress invariant) = $\sigma_1 + \sigma_2 + \sigma_3 = \sigma_d + 3\sigma_3$, K_1 , K_2 = Regression constants reflecting material type and physical state.

Various agencies and researchers have developed techniques for solving the above equation. In general the procedures can be grouped into three categories, namely: Empirical Relationships; Iterative layered approach and Finite element solutions. Each of these techniques has its own advantages and limitations when used as the basis for a rational, flexible pavement design procedure. It should be noted that each technique is geographically and environmentally sensitive. It is therefore necessary that adequate modification be made in order to develop a rationally applicable design procedure for the tropical countries.

The first purpose of this paper is to review some of these models that are based on non-linear elastic layered theory that determines the resilient response of granular materials. The second purpose is to implement these models in a flexible pavement design procedure and compare the response with field data collected from a high temperature and rainfall environment. Subsequently the response will be used to develop a model for the tropics.

METHODOLOGY

Following a comprehensive literature review, based on accuracy, simplicity and wide range of applicability, two non-linear elastic layered granular base modular predictive models were selected for further study and development, these are (1) Australian Procedure and (2) Smith and Witczak Granular Base Predictive Model. These two models are used to predict deflections, stresses and strains in pavements. The results are then correlated with field results collected from several randomly selected pavement sections in a tropical region. The one that gives the best result is selected for further modification of the algorithm in-order to reflect material non-linearity and give better fit to the resilient triaxial test data and field measured values.

Australian Procedure

This procedure was developed by the Australian Road Research Board (AUSTROADS, 1992). The procedure incorporates nonlinear material behavior directly into linear layered models through the use of an iterative-stress-modulus approach. Sublayers within the nonlinear layer are developed and assumed moduli values initially assigned to each. Layer solutions are then performed and states of stress within each sublayer are computed. These stress results are then substituted into the modulus expression obtained from the triaxial test as given by equation (1) to obtain predicted moduli results. Iterations are pursued until a tolerable error difference between these moduli is reached.

Smith and Witczak Equivalent Granular Base Predictive Model

This is a simple, practical and accurate technique that has been developed to predict the equivalent granular modulus from non linear material characterization parameters (Smith and Witczak, 1981). In this approach the layered iteration approach was accomplished to determine granular base sublayer moduli for a particular combination of pavement parameters affecting the bulk stress. Subsequently equivalent base moduli were then determined by a graphical technique that allowed the determination of a unique base modulus that resulted in the critical strain value determined from the nonlinear stress iteration process. As a result of this procedure, unique E_{2et} and E_{2ev} values were determined for a particular function of h_1 , h_2 , E_1 , E_3 , K_1 , and K_2 , where h_1 is the thickness of the asphalt concrete layer. Multiple regression technique was then utilized to obtain the predictive equations. The equations are shown below:

$$\begin{aligned} \text{Log}(E_{2et}) = & 1.079 - 0.511\text{Log}(h_a) - 0.008\text{Log}(h_b) - 0.155\text{Log}(E_a) + 0.279\text{Log}(E_s) \\ & + 0.888\text{Log}(K_1) \end{aligned} \quad (2)$$

$$\begin{aligned} \text{Log}(E_{2ev}) = & 1.105 - 0.430\text{Log}(h_a) - 0.073\text{Log}(h_b) - 0.122\text{Log}(E_a) + 0.249\text{Log}(E_s) \\ & + 0.848\text{Log}(K_1) \end{aligned} \quad (3)$$

where: E_{2et} = Equivalent Base Modulus in the horizontal direction (ksi), E_{2ev} = Equivalent Base Modulus in the vertical direction (ksi), h_a = Asphalt layer thickness (in.), h_b = Base course thickness (in.), E_a = Modulus of the Asphalt layer (ksi), E_s = Modulus of the subgrade (ksi), K_1 = Intercept of regression constant from the resilient modulus test plot (ksi), K_2 = Gradient regression constant from the resilient modulus test plot.

Field Data for Validation.

To assess the validity of the models, field data were collected from the tests sections in Nigeria, which falls in a tropical environment. . A comprehensive and reliable field and laboratory tests were performed on the entire road network of the country by the Pavement Evaluation Unit Kadanu, Nigeria in conjunction with Texas Research and Development Foundation (Claros et al, 1986). Field measurement on a factorial of thirty-six master test sections covering the observed ranges of thickness, surface, traffic and climatic variables. A careful selection from all the field and laboratory tests results performed on the thirty six master stations of the country was done to make

sure that only stations where resilient modulus tests were performed on the base were selected. The field strains values were calculated from the deflection basin matching determined from the deflectometer; and from the field and laboratory data.

RESULTS AND OBSERVATIONS

The results of strains obtained by the two models were compared with the field values. Fig.1 and Fig.2 reveal the level of closeness of the horizontal strain (fatigue strain at the bottom of the asphalt layer) and the vertical strain (rut strain at the top of the subgrade) of the predicted values and the field values. From Fig.2 it can be easily seen that the Smith-Witczak predicted values are closer to the field results than the Australian model. However the level of closeness cannot easily be distinguished in Fig.1 for horizontal strains. Consequently to accurately determine the closer of the models, significant test by using the Student-T analysis at 95 percent confidence interval was performed. From the test Smith-Witczak method gives a better prediction of the field value. The analysis shows that there is no significant difference between the field values and the predicted values, whereas for the Australian method there is a significant difference in the results obtained.

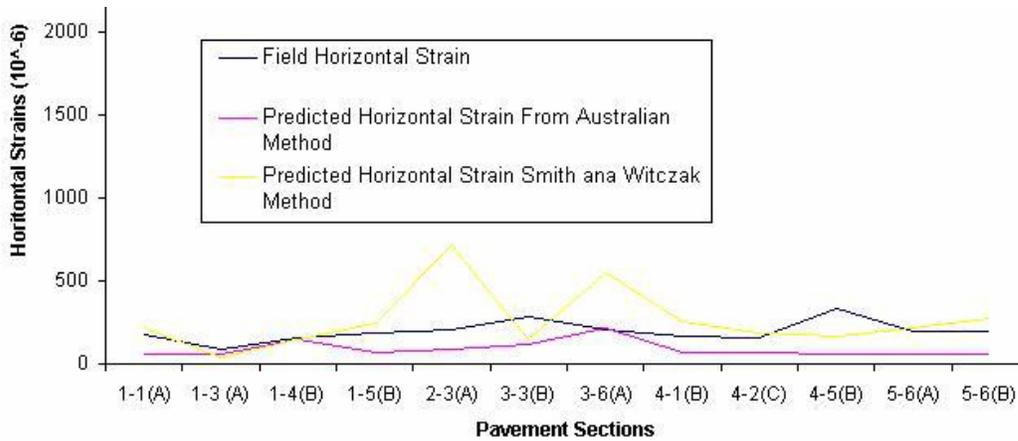


FIG. 1: Comparison of the Predicted Horizontal Strains with the Field Values

As it can be seen in Fig.1 both models give a better prediction of the horizontal strains i.e. the fatigue strain at the bottom of the asphalt layer, this is because they both give higher moduli at the top of the granular sublayer (similar to actual field values). The reason being that for classical flexible pavement structure, the modulus of the granular sublayer at the top of the base is much higher than at the bottom of the base course as a result of rapid attenuation of the bulk stress with depth. For this reason most of the predicted values for vertical strains are higher, resulting from the lower moduli obtained at the bottom of the granular layer by the two models. Additionally as shown by Fig.2 the vertical strains estimated by the Australian method are extremely high because of the very low estimated moduli for the bottom of the granular sub layer. This resulted from the fact that the constituent equations of the method (Australian) do not depend solely on the base quality K_1 and the limitation of the modular ratio to four in which other properties of the layers

above the subgrade were not taken into consideration. The Smith-Witczak method, on the other hand, gives a better estimation of the base moduli because it incorporates most of the variables that affect pavement behavior. The Smith-Witczak method was therefore adopted for fine-tuning for tropical conditions because of the above reasons. Besides, the method requires fewer computers

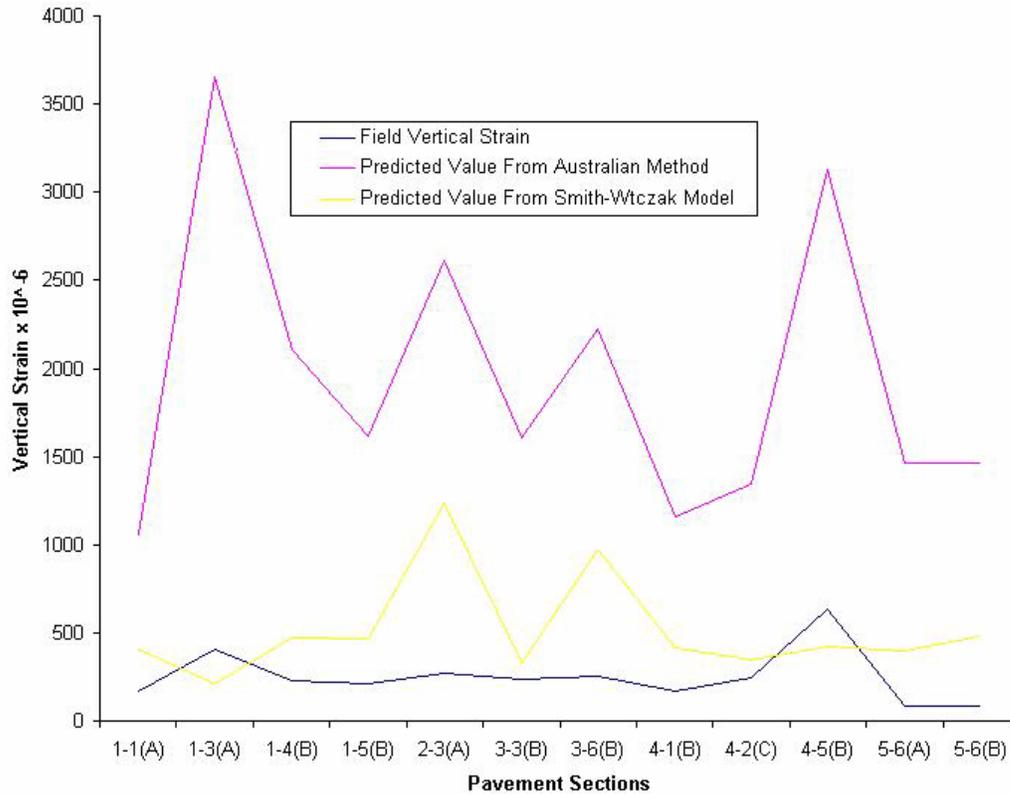


FIG. 2: Comparison of the Predicted Vertical Strains with the Field Results

time than the Australian, since the iterative elastic layered approach is not involved.

Modification of the Selected Model

After a careful observation of the material properties of the representative cross-sections studied, it was noted that there is a wide range of K_2 value for tropical granular materials, in the studied area the values range from -0.33968 to 0.40200 . This is in contrast to the small range proposed by Smith and Witczak, (Smith and Witczak, 1981) resulting from observation of the soils in the temperate environment. Consequently the assumed constant value of 0.5 for K_2 by their model may not be applicable to tropical conditions. Therefore the model was modified to accommodate the wide range of K_2 for tropical soils as revealed in this study.

To accomplish the above, “a back calculation” technique by deflection basin matching was

adopted to give the resilient base moduli value that match the fatigue and rut strains obtained from the field results. The shortfall between the predicted and actual moduli was then accounted for by the inclusion of K_2 as a variable in the equations. To incorporate the K_2 values in the Smith-Witczak model, a statistical regression analysis was performed. The shortfall between the field values and the predicted values was regressed against the K_2 values. The analysis yielded the following predictive equations for the Equivalent Base Modulus in the horizontal direction (E_{2et}) and Equivalent Base modulus in the vertical direction (E_{2ev}) respectively.

$$\begin{aligned} \text{Log}(E_{2et}) = & 1.079 - 0.511\text{Log}(h_a) - 0.008\text{Log}(h_b) - 0.155\text{Log}(E_a) + 0.279\text{Log}(E_s) \\ & + 0.888\text{Log}(K_1) + 1.257(K_2) \end{aligned} \quad (4)$$

$$\begin{aligned} \text{Log}(E_{2ev}) = & 1.105 - 0.430\text{Log}(h_a) - 0.073\text{Log}(h_b) - 0.122\text{Log}(E_a) + 0.249\text{Log}(E_s) \\ & + 0.848\text{Log}(K_1) + 1.338(K_2) \end{aligned} \quad (5)$$

where: E_{2et} = Equivalent Base Modulus in the horizontal direction (ksi), E_{2ev} = Equivalent Base Modulus in the vertical direction (ksi), h_a = Asphalt layer thickness (in.), h_b = Base course thickness (in.), E_a = Modulus of the Asphalt layer (ksi), E_s = Modulus of the subgrade (ksi), K_1 = Intercept of regression constant from the resilient modulus test plot (ksi), K_2 = Gradient regression constant from the resilient modulus test plot.

From the regression analysis, the high r^2 value of 0.70 obtained for the two determinations shows a good level of reliability on the modified algorithms. This means that about 70 percent of the total variation of the discrepancy or shortfall between the field results and the predicted value was accounted for by the inclusion of the K_2 variable. Additionally significant tests performed on the coefficient of K_2 also revealed a high level of reality.

The modified model was then implemented in a mechanistic pavement design procedure. The results obtained were very reasonable for tropical conditions and this was also used to obtain the influence of all pavement variables (layer thickness, environmental effects of temperature and moisture upon moduli, granular material quality as measured by the laboratory resilient moduli test) upon the probable performance of flexible systems throughout various periods of the year (Owolabi,1999). From the results obtained from the analysis it was found out that the most significant variable affecting the pavement performance is the K_1 parameter (quality of base course material), then followed by the subgrade modulus E_s . A more cost effective design chart was then developed for tropical regions in which the least significant variables of base thickness, asphalt layer modulus and thickness are kept at optimum constants of 150mm, 2000Mpa and 50mm respectively. The choice of 2000Mpa as asphalt modulus was chosen because of the ambient temperature experienced in the tropics. Fig. 3 shows the chart that was developed and also from the chart another model was also developed for tropical environment, which relates number of repetitions of Standard 80KN axle load to K_1 and E_s :

$$\text{Log}(N_f) = 4.016\text{Log}(K_1) + 1.4048\text{Log}(E_s) - 3.4721 \quad (6)$$

where: N_f = no. of repetitions of standard 80KN axle load, K_1 = intercept regression constant in from the modulus test MPa and E_s = Subgrade modulus in Mpa.

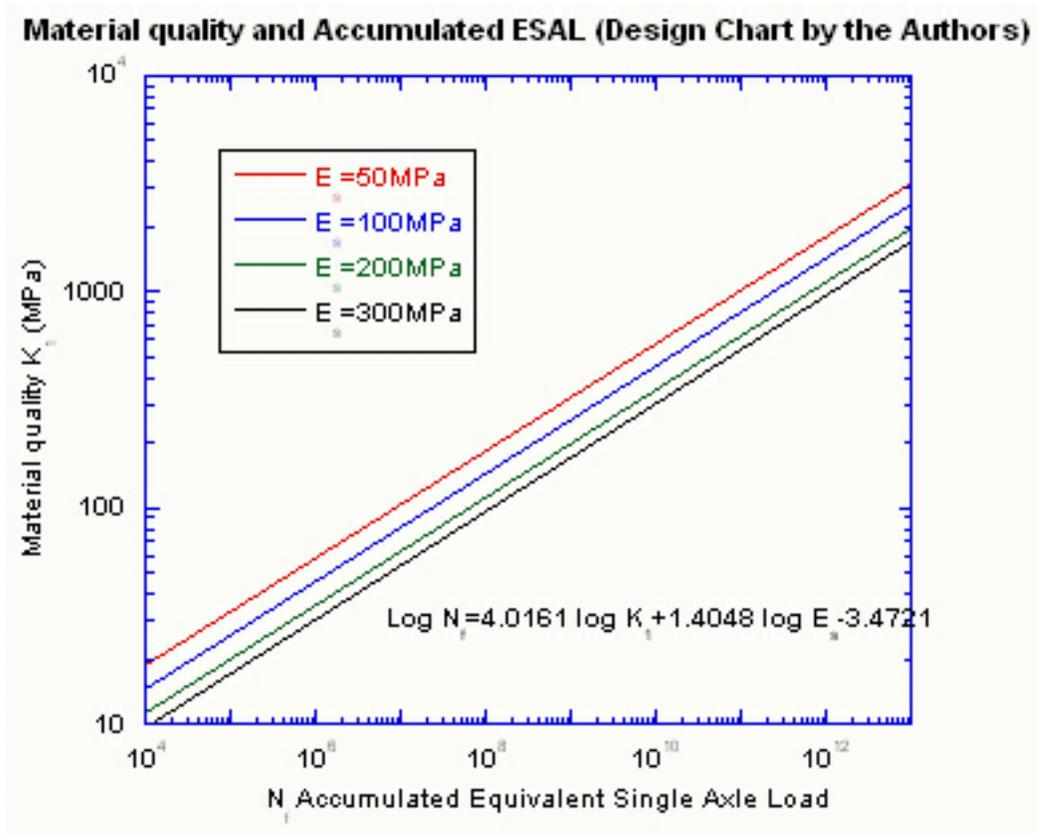


FIG. 3: Material Quality and Accumulated ESAL (Design Chart by the Authors)

CONCLUSION

The Smith and witzak base moduli predictive model is been adopted as a granular base characterization model in the tropics because of its good correlation with field results, relative simplicity, relatively shorter computational time and wide range of applicability when used in analyzing tropical soils. The shortfall observed between the field results and the predicted values from the model was mainly due to the omission of the K_2 constant from the model. The algorithms of the model was then modified by the inclusion of the K_2 in the mathematical equation. Subsequently the developed model gives a superior correlation with field results obtained in the tropics. The developed model was used to determine the influence of all pavement variables upon the probable performance of flexible systems throughout the period of the year. A cost effective design chart was also developed for tropical roads

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APPENDIX I. NOTATION

M_R = resilient modulus

σ_d = axial deviator stress

ε_a = recoverable axial strain

θ = bulk stress (first stress invariant) = $\sigma_1 + \sigma_2 + \sigma_3 = \sigma_d + 3\sigma_3$,

K_1, K_2 = Regression constants reflecting material type and physical state.

E_{2et} = Equivalent Base Modulus in the horizontal direction (ksi)

E_{2ev} = Equivalent Base Modulus in the vertical direction (ksi),

h_a = Asphalt layer thickness (in.)

h_b = Base course thickness (in.)

E_a = Modulus of the Asphalt layer (ksi)

E_5 = Modulus of the subgrade (ksi)

K_1 = Intercept of regression constant from the resilient modulus test plot (ksi)

K_2 = Gradient regression constant from the resilient modulus test plot

N_f = No. of repetitions of standard 80KN axle load